Prof. Dr. A. Chernov SS 2012

V4E2 - Numerical Simulation

Numerical Methods for Stochastic PDEs

Description: Many practical problems in computation science can be written in a form

$$\mathcal{A}u = f$$

where $\mathcal{A}$ is a linear or nonlinear operator representing the system behavior, u is the value of interest and f are input data such as boundary conditions of the volume force. Due to permanent growth of computer power in the recent decades large numerical simulations became possible providing very high quality numerical approximation of u provided $\mathcal{A}$ and f are known exactly. In practical applications, however, the exact form and the structure of $\mathcal{A}$ and f entering the computations are based on a *mathematical model* being itself only an approximation for the actually occurring process. Hence, there is no point in increasing numerical accuracy beyond the modeling error or the error in the measurements.

One way to overcome this difficulty is to treat the lack of knowledge or the measurement error using probabilistic description, which might be viewed as a more realistic and powerful model for the underlying process. In this case, input data f, coefficients of the operator $\mathcal{A}$ or even the form of the computational domain can be modeled as random fields leading to *stochastic partial differential* or *integral equations*, which solution u becomes a random field as well. This new class of problems involving random variables might be viewed as a parametric PDE/integral equation, which usually leads to high dimensional formulations. In this case, standard discretization methods are too expensive and can not be applied in practice. Hence, new numerical techniques are needed.

In this lecture course we give an overview of the existing numerical solution techniques for such problems. We concentrate on the Galerkin discretization in physical domain combined with various treatment for the stochastic variables, such as

- Tensorization Techniques
- Karhunen-Loève approximation
- Homogeneous Chaos

The lecture is addressed to Diploma, Master and PhD students.

Requirements: Numerical Algorithms - Wissenschaftliches Rechnen I - III or similar, basic knowledge of Probability Theory.

Place and time: Lecture: Tue, Thu 10:15–12:00, Exercises: Fri 14:15–16:00, weekly, LWK Altbau (Endenicher Allee 60), room 1.007

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References

[1] Christoph Schwab and Claude Jeffrey Gittelson. Sparse tensor discretizations of high-dimensional parametric and stochastic PDEs. *Acta Numer.*, 20:291–467, 2011.

[2] Roger G. Ghanem and Pol D. Spanos. *Stochastic finite elements: a spectral approach.* Springer-Verlag, New York, 1991.

[3] Alexey Chernov and Christoph Schwab. Sparse p-version BEM for first kind boundary integral equations with random loading. *Appl. Numer. Math.*, 59(11):2698–2712, 2009.

[4] Helmut Harbrecht, Reinhold Schneider, and Christoph Schwab. Sparse second moment analysis for elliptic problems in stochastic domains. *Numer. Math.*, 109(3):385–414, 2008.

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