

Summer semester 2013

# Graduate Seminar on Efficient Simulation

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## 1 Summary

The numerical solution of partial differential equations (PDEs) requires advanced techniques to achieve optimal or near-optimal performance on realistic and large-scale problems. We will discuss such methods for efficient numerical simulation, divided into two topics:

**Geometric and algebraic multilevel methods:** How to construct fast preconditioners for solving elliptic PDEs?

**Spectral and spectral element methods:** Techniques for high-order discretizations of elliptic/hyperbolic PDEs.

Basic knowledge on interpolation, numerical quadrature, finite element methods for elliptic PDEs, and multigrid methods will be advantageous.

The seminar presentations (50 minutes) should be self-consistent and understandable without requiring specialized prior knowledge. A four-page written summary in L<sup>A</sup>T<sub>E</sub>X must be turned in by email or in print until July 26, 2013, with a border of 2 cm and 11 pt font size.

The seminar will take place on Mondays at 3:00pm s.t. in room 5.002, Wegelerstr. 6.

## 2 Topics

1. Preconditioners, smoothers, and relaxation methods
2. The multigrid method for elliptic PDEs
3. The multigrid method for the advection-diffusion equation
4. The multigrid method for nonlinear equations and FAS
5. The algebraic multigrid (AMG) method
6. Parallelization of the AMG method
7. Black-box multigrid
8. Introduction to spectral methods
9. Fourier spectral methods
10. Legendre and Chebyshev methods
11. The spectral element method
12. The discontinuous Galerkin method
13. Parallel adaptive high-order methods

## References

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- [4] W. HACKBUSCH, *Multigrid Methods and Applications*, vol. 4 of Springer Series in Computational Mathematics, Springer, 1985.
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