Hierarchical Matrices

Summer semester 2013
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Exercise Sheet 2.

Due date: **Thursday**, **09.05.**

Please send your source code and plots to gesenhues@ins.uni-bonn.de

Exercise 1. (Programming: PCA)

The objective of this exercise is to subdivide a set of points $X_t := \{x_i \in \mathbb{R}^3, i \in t\}$ into two subsets indicated by t_1 and t_2 using principal component analysis as introduces in the lecture. In the case of points, $z_i = x_i$ and $\mu(x_i) = 1$.

Let $t := \{1, \ldots, n\}$. Generate n random three-dimensional points and implement a method to subdivide them using PCA. Give a permutation array to reorder the indices in t to comply with the remark after Lemma 1.14.

Exercise 2. (Programming: Admissibility condition)

Let $f \colon \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ with $f(x,y) = 1/\left\| x - y \right\|_2$. For $x_i \in X$, $y_j \in Y$, $i, j = 1, \ldots, n$, define $F_{i,j} := f(x_i, y_j)$.

   a) Write a function that generates n random points in a cuboid $C_{a,b,c}(x,y,z) := [x - a, x + a] \times [y - b, y + b] \times [z - c, z + c]$.

   b) Let $X \subset C_{1/2,1/2,1/2}(x_t, 0, 0)$ and $Y \subset C_{1/2,1/2,1/2}(0, 0, 0)$. Compute the SVD of F for $n = 1000$ and $x_t = 20, 10, 2, 1$ and plot the first 100 singular values using a semi-log plot. What do you observe? Use BLAS or LAPACK.