Hierarchical Matrices

Summer semester 2013
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Exercise Sheet 4.

Due date: **Friday**, **31.05.**

Exercise 1. (Another Inversion Method)

Instead of using the recursive algorithm from chapter 2.6, it is also possible to invert a Hierachical Matrix in the following way:
Let $A \in \mathcal{H}(T_{I\times I}, k)$ be invertible. The inverse of A solves the nonlinear equation $f(X) := A - X^{-1} = 0$. Because f is differentiable, the Newton method is appliable.

a) Derive the iteration rule for the Newton method.

b) Let $X^{(0)}$ be a start value satisfying $\|A\| \left\|X^{(0)} - A^{-1}\right\| =: q < 1$, where $\|\cdot\|$ is a sub-multiplicative matrix norm. Show the quadratic convergency

$$\left\|X^{(m)} - A^{-1}\right\| \leq q^{2^m} \left\|A^{-1}\right\|.$$

c) Let A be positive-definite. Let $X^{(0)}$ be chosen so that both $X^{(0)}$ and $A^{-1} - X^{(0)}$ are positive-definite. Show global convergence and that $X^{(m)}$ is positive-definite for every m.

Hint: For $F_m := I - A^{1/2}X^{(m)}A^{(1/2)}$ positive-definite show that $F_{m+1} = F_m^2$.

d) What is the downside of this method compared to the one from the lecture?