



## Hierarchical Matrices

Summer semester 2013  
Prof. Mario Bebendorf  
Jos Gesenhues



### Exercise Sheet 5.

Due date: **Thursday, 06.06.**

---

#### Exercise 1. (Jump relations)

Let  $\Gamma$  be smooth. For  $y \in \mathbb{R}^3$  let

$$(\mathcal{V}w)(y) := \frac{1}{4\pi} \int_{\Gamma} \frac{w(x)}{\|x-y\|} ds_x,$$
$$(\mathcal{K}w)(y) := \lim_{\varepsilon \rightarrow 0} \frac{1}{4\pi} \int_{x \in \Gamma: \|x-y\| \geq \varepsilon} w(x) \partial_{\nu_x} \frac{1}{\|x-y\|} ds_x$$

be the single and the double-layer operator, respectively. Validate the formulas

$$\partial_{\nu}(\mathcal{V}w)(y) \rightarrow (\mathcal{K}'w)(y_0) \pm \frac{1}{2}w(y_0)$$
$$(\mathcal{K}w)(y) \rightarrow (\mathcal{K}w)(y_0) \mp \frac{1}{2}w(y_0)$$

for  $y \rightarrow y_0 \in \partial\Omega$  and  $y \in \Omega$ . Here,

$$(\mathcal{K}'w)(y) := \lim_{\varepsilon \rightarrow 0} \frac{1}{4\pi} \int_{x \in \Gamma: \|x-y\| \geq \varepsilon} w(x) \partial_{\nu_y} \frac{1}{\|x-y\|} ds_x$$

denotes the adjoint double-layer operator.

#### Exercise 2. (Singularity function)

Show that

$$S(x) = \frac{\exp(i\sqrt{\alpha}\|x\|)}{4\pi\|x\|}$$

is the singularity function for the three-dimensional Helmholtz operator.