Hierarchical Matrices

Summer semester 2013
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Exercise Sheet 6.

Due date: **Tuesday**, **18.06.**

Exercise 1. (Multipole Expansion)

Prove the error estimate

$$\left| \frac{1}{\|x-y\|} - \kappa_p(x,y) \right| \leq \frac{1}{\|x\| - \|y\|} \left(\frac{\|y\|}{\|x\|} \right)^p,$$

where $\|x\| > \|y\|$ and

$$\kappa_p(x,y) = \frac{1}{\|x\|} \sum_{\ell=0}^{p-1} \left(\frac{\|y\|}{\|x\|} \right)^\ell P_\ell(\hat{x} \cdot \hat{y})$$

is the multipole expansion of the Coulomb potential, P_ℓ is the ℓth Legendre polynomial and $\hat{x} = x/\|x\|$, $\hat{y} = y/\|y\|$.
Hint: Legendre polynomials can be defined as:

$$\frac{1}{\sqrt{1-2st+t^2}} = \sum_{\ell=0}^{\infty} P_\ell(s) t^\ell.$$

Exercise 2. (Asymptotic Smoothness)

Prove that the product of two asymptotically smooth functions is asymptotically smooth. Prove that the sum of two positive asymptotically smooth functions is asymptotically smooth.

Exercise 3. (Variant of Lemma 3.2)

Prove the Remark after Lemma 3.2.