Hierarchical Matrices

Summer semester 2013
Prof. Mario Bebendorf
Jos Gesenhues

universität**bonn**

Exercise Sheet 7.

Due date: **Tuesday**, **25.06.**

Exercise 1. (Another Kind of Approximation)

Let

$$s_k(x, y) = \kappa(x, [y]_k)^T W_k^{-1} \kappa([x]_k, y)$$

be the kth order approximation of $\kappa(x, y)$ from the lecture.

Prove the representation

$$s_k(x, y) = \sum_{\ell, m=1}^{k} (-1)^{\ell+m} \kappa(x, y_\ell) \kappa(x_m, y) \frac{\det W_k^{(m,\ell)}}{\det W_k},$$

where $W_k^{(\ell,m)} \in \mathbb{R}^{(k-1) \times (k-1)}$ is W_k without the ℓth row and the mth column.

Exercise 2. (Rank Reduction)

Let $A \in \mathbb{C}^{m \times n}$. Prove that for any $1 \le i \le m$, $1 \le j \le n$ with $A_{ij} \ne 0$,

$$\mathrm{rank}(A - A_{:,j} A_{ij}^{-1} A_{i,:}) \le \mathrm{rank}(A) - 1.$$