Hierarchical Matrices

Summer semester 2013
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Exercise Sheet 8.

Due date: **Tuesday, 02.07.**

Exercise 1. (ACA)

Let $R_0 \in \mathbb{R}^{m\times n}$ and

$$R_{k+1} = R_k - \frac{(R_k)_{1:m,j_{k+1}}(R_k)_{i_{k+1},1:n}}{(R_k)_{i_{k+1}j_{k+1}}},$$

where i_{k+1} and j_{k+1} are chosen according to

$$|(R_k)_{i_{k+1}j_{k+1}}| = \max_{i=1,\ldots,m} |(R_k)_{ij_{k+1}}| \neq 0.$$

Note: This is a transposed definition compared to the lecture.

a) Show that there are permutation matrices $P \in \mathbb{R}^{m\times m}$ and $Q \in \mathbb{R}^{n\times n}$ with

$$(PR_kQ)_{ij} = 0, \quad i,j = 1,\ldots,k.$$

b) Show that

$$R_k = (A - (AQ)_{1:m,1:k}X) - \Xi_k\left((PA)_{1:k,1:n} - (PAQ)_{1:k,1:k}X\right)$$

for all $X \in \mathbb{R}^{k\times n}$, where $\Xi_k := (AQ)_{1:m,1:k}(PAQ)^{-1}_{1:k,1:k}$.

c) Argue why ACA is mathematically equivalent to the LU-decomposition with partial pivoting of the matrix AQ. What is the difference between the two?

d) Show $\|\Xi_k\|_\infty \leq 2^k - 1$.