Institut für Numerische Simulation
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Computer lab
Numerical Methods for Thin Elastic Sheets
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Problem sheet 2 **May 27th, 2013**

Let $\mathcal{T}_h$ be a regular triangulation of a two dimensional surface $\Omega \subset \mathbb{R}^3$ and $T \in \mathcal{T}_h$ denote a triangle with corner points $b_1, b_2, b_3 \in \mathbb{R}^3$ and edge midpoints $b_4, b_5, b_6 \in \mathbb{R}^3$.
Instead of considering the space linear of Finite Elements, you shall now study the space of quadratic Finite Elements

$$\mathcal{V}_h := \{v \in C^0(\Omega) \mid v|_T \in \mathcal{P}_2(T)\ \forall T \in \mathcal{T}_h\},$$

where $\mathcal{P}_2(T)$ is the space of quadratic polynomials over T. Furthermore, let $\mathcal{B}_h := \{\phi_1, \phi_2, \dots\}$ denote a basis of $\mathcal{V}_h$.

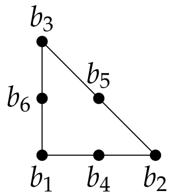

Figure 1: Quadratic Lagrange Triangle

(i) Compute the standard Lagrange basis of $\mathcal{V}_h$ w.r.t b_i $(i = 1, \dots, 6)$ in barycentric coordinates.

(ii) Compute the mass matrix and stiffnes matrix in local coordinates with $\mathcal{P}_2$ elements:

$$M_{ij} = \int_T \phi_i(x)\phi_j(x)da,$$
$$S_{ij} = \int_T \nabla\phi_i(x)\nabla\phi_j(x)da.$$

(iii) Implement $\mathcal{P}_2$ Finite Elements in your project directory. Therefore, copy the file *labsheetTemplates/labsheet2/quadraticFE.h* and implement the missing parts. Furthermore, consider the problems of *labsheet 1* with $\mathcal{P}_2$ elements and compare the approximation error of $\mathcal{P}_2$ elements with the approximation error of $\mathcal{P}_1$ elements. Are your observations consistent with the theory?