# Numerical Simulation

Summer semester 2014
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## Exercise Sheet 10.

Due date: **Tuesday, 15 July**.

**Exercise 13.** The function $y : (x, t) \mapsto x^{-1/2} e^t$ is in the space $C(0, T, L^1(0, 1))$. Compute its norm. Which spaces $L^p(0, T, L^q(0, 1))$ does it also belong to?

(4 points)

**Exercise 14.** Consider the control problem

$$\min J(y, u) := \frac{1}{2} \|y(T) - y_\Omega\|^2_{L^2(\Omega)} + \frac{\lambda}{2} \|w\|^2_{L^2(\Omega)}$$

with $w \in L^2(\Omega)$, $|w(x)| \leq 1$ almost everywhere in $\Omega$ and

$$\begin{aligned} y_t - \Delta y &= 0 \quad \text{in } \Omega \\ \partial_\nu y + y &= 0 \quad \text{in } \Sigma \\ y(0) &= w \quad \text{in } \Omega. \end{aligned}$$

We assume that $\Omega \subset \mathbb{R}^N$ is a bounded Lipschitz domain with boundary $\Gamma$ and

$$\Sigma = \Gamma \times (0, T), \quad y_\Omega \in L^2(\Omega), \quad \lambda \geq 0.$$

Prove existence of an optimal control and derive necessary optimality conditions.

(8 points)