



## Numerical Simulation

Summer semester 2014  
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### Exercise Sheet 2.

Due date: **Tuesday, 29 April.**

**Exercise 2.** Let  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  be  $C^2$ -continuous. Prove that

- a)  $\nabla_x^2 f$  is self-adjoint and
- b)  $\nabla_x^2 f u = \nabla_x (\nabla_x f, u)$  for all  $u \in \mathbb{R}^d$ .

(6 points)

**Exercise 3.** Consider two Hilbert spaces  $\{Y, \|\cdot\|_Y\}$  and  $\{U, \|\cdot\|_U\}$  with  $y_d \in Y$ ,  $\lambda \geq 0$  and some linear continuous operator  $S : U \rightarrow Y$ . Prove strict convexity of the functional

$$f(u) = \|Su - y_d\|_Y^2 + \lambda \|u\|_U^2,$$

provided that  $S$  is injective *or*  $\lambda > 0$ .

(4 points)