



## Numerical Simulation

Summer semester 2014  
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### Exercise Sheet 3.

Due date: **Tuesday, 6 May.**

**Exercise 4.** Given  $y_0 \in V$ , consider  $y_1, \dots, y_K \in V$  arisen from the discretized PDE

$$y_{k+1} = y_k + \Delta t (f_k - (u_1 M^{-1} A + u_2) y_k), \quad k = 0, \dots, K-1. \quad (0.5.31)$$

a) Identify  $T(y, u) = G_u y - F$  with block matrix  $G_u : V^{K+1} \rightarrow V^{K+1}$  and block vector  $F \in V^{K+1}$ .

b) With

$$J(y, u) = \frac{1}{2} \|y - y_d\|_{M,t}^2 + \frac{\lambda}{2} \|u\|_{\mathbb{R}^2}^2, \quad (0.5.32)$$

formulate the Lagrangian using  $p_0, \dots, p_K \in V$ . Interpret  $p_k$  as coefficient vector analogous to  $y_k$  and use an appropriate inner product.

c) Derive the adjoint equation. Write it in the form of (0.5.31). What do you observe?

d) Compute two components of the reduced gradient.

(9 points)