Exercise Sheet 4.

Due date: **Tuesday, 13 May**.

Exercise 5. Let $c_0 \in L^\infty(\Omega)$ and $\alpha \in L^\infty(\Gamma)$ be non-negative almost everywhere with

$$\int_\Omega c_0(x)^2 \,\mathrm{d}x + \int_\Gamma \alpha(x)^2 \,\mathrm{d}x > 0.$$

Then the boundary value problem

$$\begin{aligned} -\Delta y + c_0 y &= f \quad \text{in } \Omega, \\ \partial_r y + \alpha y &= g \quad \text{on } \partial\Omega. \end{aligned} \tag{1.1.7}$$

has a unique weak solution y for any data $f \in L^2(\Omega)$ and $g \in L^2(\Gamma)$.

(6 points)

Exercise 6. Consider an arbitrary Hilbert Space $(H, (\cdot,\cdot))$, a weakly convergent sequence $u_n \rightharpoonup u$ and a strongly convergent sequence $v_n \to v$.
Prove convergence of the scalar products $(u_n, v_n) \to (u, v)$.

(3 points)