Numerical Simulation

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Exercise Sheet 5.

Due date: **Tuesday, 20 May**.

Exercise 7. Prove that the following functionals are Fréchet differentiable.

a) $f(u) \coloneqq \sin(u(1))$ for $u \in C([0,1])$

b) $f(u) \coloneqq \|u\|_H^2$ for u in some Hilbert space $(H, (\cdot,\cdot))$

c) $f(u) \coloneqq \Delta(u^2)$ for $u \in H^2$.

(6 points)

Exercise 8. Show that in finite-dimensional spaces, continuity of the Gâteaux derivative implies total differentiability in the classical sense.

(3 points)

Exercise 9. Knowing that $L^1 \subset (L^\infty)^*$, show that the operator

$$R : L^1(\mathbb{R}) \to L^\infty(\mathbb{R}), \quad f \mapsto \int_{\mathbb{R}} f \,\mathrm{d}x \ \text{(constant function)}$$

is self-adjoint for L^1-functions and discuss the adjoint operator for $(L^\infty)^*$-functions.

(3 points)