Scientific Computing II

Summer Semester 2014
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Excercise sheet 2.

Closing date **22.04.2014**.

Theoretical exercise 1. (Residual error estimator [5 points])

Let $\Omega \subset \mathbb{R}^2$ be a bounded, open and connected domain with Lipschitz boundary and cone condition. Let $a(v, w) = \int_\Omega \nabla v \cdot \nabla w \, dx$ for $v, w \in \mathbb{V} = H_0^1(\Omega)$ and let F be a linear functional on $\mathbb{V}$. Let $\mathbb{V}_h \subset \mathbb{V}$ be a FE-subspace. It holds $a(u, v) = F(v)$ for all $v \in \mathbb{V}$ and $a(u_h, v) = F(v)$ for all $v \in \mathbb{V}_h$. Show that

$$a(u - u_h, v) = \sum_{i=1}^{nel} \left(\int_{\tau_s} r_h v dx + \frac{1}{2} \sum_{E \in \tau_s \setminus \partial\Omega} \int_E R_h v ds \right) \quad \forall\, v \in \mathbb{V}$$

where τ_s denotes one of the nel elements and E denotes an edge. Here, r_h denotes the residual on the corresponding element and R_h denotes the jump of the normal derivative of u_h.

Theoretical exercise 2. (Higher order elements [5 points])

Draw an example for an irregular mesh with one hanging node and give a condition on the value of a finite element function on this node to keep the discretization conformal (C_0) for the case of

a) quadratic Lagrange elements and

b) cubic Lagrange elements.

Theoretical exercise 3. (2nd order PDE with jump at the boundary [5 points])

For Ω as in exercise 1 provide a residual error estimator for a second order elliptic PDE

$$-\mathrm{div}(A(x)\nabla u(x)) + c(x)u(x) = f(x) \quad \text{on } \Omega$$

with Dirichlet zero boundary conditions where the 2×2-matrix A is constant on each element.

Theoretical exercise 4. (Lamé-equation [5 points])

Let Ω be a domain as in exercise 1. Derive a residual error estimator for the stationary Lamé equation

$$-\mu\vec{u} - (\lambda + \mu)\,\mathrm{grad}\,\mathrm{div}\vec{u} = \vec{0}$$

with Dirichlet zero boundary conditions on $\Gamma_1 \subset \partial\Omega$ and constant Neumann boundary condition with value c on $\Gamma_2 = \partial\Omega \setminus \Gamma_1$. Use the weak formulation with a linear finite element space $\mathbb{V}_h \subset \mathbb{V} = \left[H_0^1(\Omega)\right]^3$.