Scientific Computing II

Summer Semester 2014
Lecturer: Prof. Dr. Beuchler
Assistent: Bastian Bohn

universitätbonn

Excercise sheet 4.

Closing date **06.05.2014**.

Theoretical exercise 1. (Bisection of an isosceles triangle [5 points])

For the triangle Δ with interior angles ρ, τ, σ let $0 < \tau \leq \frac{\pi}{3}$ and $\rho = \sigma = \frac{\pi}{2} - \frac{\tau}{2}$. Bisect the triangle along the longest edge. Show that the smallest angle x_τ which occurs in the two new triangles fulfills

$$\tan x_\tau = \frac{\sin \tau}{2 - \cos \tau}$$

and prove that this term is larger or equal to $\tan(\frac{\tau}{2})$.

Theoretical exercise 2. (Equivalence classes under longest edge bisection [5 points])

For a triangle Δ with angles $0 < \tau \leq \sigma \leq \rho$ we denote it's similarity class by (ρ, σ, τ) (the order does not matter). When bisecting along the longest edge we get two new triangles belonging to two (possibly) new similarity classes $(\rho_i, \sigma_i, \tau_i)$ for $i = 1, 2$. To indicate the relation between Δ and the two new triangles we write

$$(\rho_1, \sigma_1, \tau_1) \leftarrow (\rho, \sigma, \tau) \rightarrow (\rho_2, \sigma_2, \tau_2).$$

a) Prove that

$$\begin{array}{ccc} (\rho, \sigma, \tau) & \leftrightarrows & (x, \tau, \rho + \sigma - x) \\ \downarrow & & \downarrow \\ (\rho - x, \sigma, x + \tau) & & (x, \rho - x, \pi - \rho) \end{array}$$

for some suitable angle x.

b) Let $\pi - \rho \geq \rho - x$. Let further one of the following conditions hold:

(i) $x + \tau \geq \sigma$ and $x + \tau \geq \rho - x$,

(ii) $\rho - x < \tau$.

Prove that

$$\begin{array}{ccc} (\rho, \sigma, \tau) & \leftrightarrows & (x, \tau, \rho + \sigma - x) \\ \uparrow\downarrow & & \uparrow\downarrow \\ (\rho - x, \sigma, x + \tau) & \leftrightarrows & (x, \rho - x, \pi - \rho) \end{array}$$

Theoretical exercise 3. (Smallest angle after triangle bisection [5 points])

For a triangle Δ with smallest angle τ bisect the triangle along its longest edge, bisect the two new triangles along their longest edges and iterate this process arbitrarily often. Prove that for any created triangle the smallest angle in that triangle is larger or equal to $\frac{\tau}{2}$.

Theoretical exercise 4. (Bubble functions in three dimensions [5 points])

Let $\hat{T} := \{(x, y, z) \in \mathbb{R}^3 \mid x, y, z \geq 0 \text{ and } x + y + z \leq 1\}$ the reference tetrahedron. Specify the interior and the face bubble functions for $\hat{T}$.