Numerical Simulation

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Excercise sheet 2.

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Theoretical exercise 1. (Frechet Derivatives [6 points])

Show that the function f is Frechet differentiable and calculate the Frechet derivatives for the following cases.

a) Let X be a real Hilbert space:

$$f(u) := \|u\|_X^2.$$

b) Let X be a real Hilbert space, $A : X \to X$ a linear, bounded and self-adjoint operator. Let furthermore $b \in X$, $c \in \mathbb{R}$:

$$f(u) := \frac{1}{2}\langle u, Au\rangle_X - \langle b, u\rangle_X + c.$$

c) Let $X = C^1([0,1])$ and let $L : [0,1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be C^1:

$$f(u) := \int_0^1 L(t, u(t), u'(t))\mathrm{d}t.$$

Theoretical exercise 2. (Equivalence for constrained equation system [7 points])

Let $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^n$ be fixed. Prove that the following are equivalent:

(i) $A^T x = b$ has a solution $x \in \mathbb{R}^m$ with $x \geq 0$.

(ii) For all $y \in \mathbb{R}^n$ with $Ay \geq 0$ it holds $b^T y \geq 0$.

The vector inequalities are to be understood componentwise.

Theoretical exercise 3. (Duality for best approximation [7 points])

Let X be a real Banach space, let $M \subset X$ be a linear subspace and let $b \in M$ be fixed. Define $M^\perp := \{f \in X^* \mid f(u) = 0\ \forall\, u \in M\}$ and $B(M^\perp) := \{f \in M^\perp \mid \|f\| \leq 1\}$. Prove that

$$\sup_{f\in B(M^\perp)} f(b) =: D$$

has a maximizer in $B(M^\perp)$ and $D = \inf_{u\in M} \|b - u\|$.