Numerical Simulation

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Excercise sheet 3.

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Theoretical exercise 1. (Tangent Vectors [6 points])

Recall definition 1.27 from the lecture: Let U be a Banach space, $u_0 \in M \subseteq U$. Then $h \in U$ is called tangent vector to M at u_0 if there exist $\epsilon > 0$ and an admissible curve $\gamma : [0, \epsilon) \to U$ such that $\gamma(t) = u_0 + th + o(t) \in M$ for $t \in [0, \epsilon)$.
Prove that the set of all tangent vectors to M at u_0 is a closed cone.

Theoretical exercise 2. (Krein's Extension Theorem [6 points])

Let U be a Banach space, let $K \subseteq U$ be a convex cone and let $L \subseteq U$ be a linear subspace of U such that $L \cap \mathrm{int}(K) \neq \emptyset$. Let furthermore $f : L \to \mathbb{R}$ be a continuous linear functional such that $f(u) \geq 0 \; \forall \, u \in L \cap K$.
Prove that f can be extended to a continuous linear functional $\bar{f} : U \to \mathbb{R}$ such that $f(u) \geq 0 \; \forall \, u \in K$.

Theoretical exercise 3. (Minimization of motion [6 points])

Let $A : C^1([-1,1]) \to \mathbb{R}$ and $B : C^1([-1,1]) \to \mathbb{R}$ be defined by

$$A(f) := \int_{-1}^{1} \sqrt{1 + f'(x)^2} \mathrm{d}x \quad \text{and} \quad B(f) := \int_{-1}^{1} f(x) \mathrm{d}x.$$

Consider the constrained minimization problem

$$A(f) \to \min! \quad \text{s.t.} \quad B(f) = \frac{\pi}{2}, f(-1) = f(1) = 0.$$

Show that Theorem 1.32 from the lecture is applicable and calculate the Lagrangian. Prove that there exist Lagrange multipliers such that $f^*(x) := \sqrt{1 - x^2}$ is a root of the derivative of the Lagrangian.