Numerical Simulation

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Exercise Sheet 8   Closing date **June 28, 2016**.

Exercise 1. Let $\Omega = (0,1)$ and $U = L^2(\Omega)$. Consider

$$f(u) := - \int_0^1 \cos(u(x))\, dx$$

Prove the following:

a) f is not twice Fréchet-differentiable in $\bar{u} = 0$ with respect to the L^2-norm.

b) f is twice Fréchet-differentiable in $\bar{u}$ with respect to the L^∞-norm.

(10 points)

Exercise 2. Use the formal Lagrange technique to derive the first order necessary optimality conditions for the Dirichlet-control problem

$$\min_\Omega \frac{1}{2}(y - y_\Omega)^2 dx + \frac{\lambda}{2} \int_\Gamma u^2 ds$$

s.t.

$$
\begin{aligned}
-\Delta y &= 0 \quad \text{in } \Omega \\
y &= u \quad \text{on } \Gamma \\
-1 \leq u(x) &\leq 1.
\end{aligned}
$$

(10 points)