Numerical Simulation

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Exercise Sheet 9

Closing date **July 5, 2016**.

Exercise 1. Let $\Omega \subset \mathbb{R}^3$ be a bounded Lipschitz domain. Prove existence of an optimal control for the following nonconvex optimal control problem:

$$\min_{(u,y)\in U\times V} J(u,y) = \frac{1}{2}(y-y_\Omega)^2 dx + \frac{\lambda}{2}\int_\Omega u^2 dx$$

s.t.

$$\begin{aligned} -\Delta y + uy &= f \quad \text{in } \Omega \\ y &= u \quad \text{on } \Gamma \\ a \leq u(x) &\leq b \text{ for a.a. } x \in \Omega, \end{aligned}$$

with $U = L^2(\Omega)$, $V = H_0^1(\Omega)$, $f, y_\Omega \in L^2(\Omega)$, $\lambda > 0$, $a \in \mathbb{R}_0^+$, $b \in \mathbb{R} \cup \{\infty\}$, $a \leq b$.

(10 points)

Exercise 2. Let $G\colon L^s(\Omega) \to H^1(\Omega) \cap C(\bar{\Omega})$, $s > n-1$, denote the control-to-state operator for the boundary value problem

$$\begin{aligned} -\Delta y + y &= 0 \quad \text{in } \Omega \\ \partial_\nu y + b(x,y) &= u \quad \text{on } \Gamma, \end{aligned}$$

under Assumptions A1-A5 formulated in the lecture. Prove that G is Fréchet-differentiable from $L^s(\Omega) \to H^1(\Omega) \cap C(\bar{\Omega})$ and state the concrete form of $G'(u)$.

(10 points)