To show iii) superlinear convergence, we observe that for any $\alpha^{\circledast}$ which is a minimizer associated with $F(v^{\circledast})$ [$\alpha^{\circledast} \in A_{v^{\circledast}}$ in the notation of the exercise] it holds

$$F(v) \le \overbrace{F(v^{\circledast}) - \left(B(\alpha^{\circledast})v^{\circledast} - C(\alpha^{\circledast})\right)}^{=0} + \underbrace{\left(B(\alpha^{\circledast})v - C(\alpha^{\circledast})\right)}_{\substack{F(v)\le \\ \text{since } \alpha^{\circledast} \text{ might not} \\ \text{be minimizer associated} \\ \text{with } F(v)}}$$

$$= B(\alpha^{\circledast})(v - v^{\circledast}).$$

In particular

$$F(v^k) \le B(\alpha^{\circledast})(v^k - v^{\circledast}) \qquad (*)$$

Generally, we have by the definitions of α^{k+1} and v^k

$$B(\alpha^{k+1})v^k - C(\alpha^{k+1}) = F(v^k)$$

$$B(\alpha^{k+1})v^{k+1} - C(\alpha^{k+1}) = 0$$

Therefore, $B(\alpha^{k+1})(v^k - v^{k+1}) = F(v^k)$

and thus

$$v^{k+1} = v^k - B(\alpha^{k+1})^{-1}F(v^k)$$

[This btw. can be seen as an iteration of a semismooth Newton method, where $B(\alpha^{k+1})$ plays the role of a derivative of F at v^k. In difference to "normal" Newton the set of minimizers is not necessarily unique]

Since $B(\alpha^{k+1})$ is monotone we get with the above bound $(*)$

$$\ge v^k - B(\alpha^{k+1})^{-1}B(\alpha^{\circledast})(v^k - v^{\circledast})$$

and thus

$$0 \ge v^{k+1} - v^{\circledast} \ge \left(I - B(\alpha^{k+1})^{-1}B(\alpha^{\circledast})\right)(v^k - v^{\circledast}) \qquad (**)$$

From the exercises, we can obtain that one can find $a^{k,\circledast}$ which is a minimizer associated with $F(v^{\circledast})$, such that $[\alpha^{k,\circledast} \in A_{v^{\circledast}}]$

$$B(\alpha^{k+1}) - B(a^{k,\circledast}) \to 0 \quad k \to \infty$$

With that $\alpha_i^{k,\circledast}$ and the monotonicities of the $B(\alpha)$ we get

$$I - B(\alpha^{k+1})^{-1} B(\alpha^{k,\circledast}) \underset{k \to \infty}{\longrightarrow} 0$$

(also using $B^{-1}(\cdot)$ is cont. on A).

Then, from $(**)$, which surely holds for $\alpha^{k,\circledast}$ as well, we obtain

$$0 \geq v^{k+1} - v^{\circledast} \geqq d(v^k - v^{\circledast})$$

which observing the signs and increasing $(v^k)_k$ proofs the superlinear convergence

$\square$