Scientific Computing 2

Summer term 2017
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Sheet 6

Submission on **Thursday, 1.6.2017**.

Exercise 1. (electrical network)

In a complex electrical network, the strength of the electric current is to be maximized via callibrating two electrical resistors $R_1, R_2 \in (0, R_{max})$. There is no mathematical model available, therefore a simple strategy is used: For a fixed R_2, we optimize over R_1. with this new R_1 fixed, we optimize over R_2. We repeat this procedure until we arrive in a fixed point of this iteration.
Is it possible to find the solution of this optimization problem with the described method? Why/Why not?

(4 points)

Exercise 2. (straight lines)

Let $f : \mathbb{R}^n \to \mathbb{R}$ be two times continuously differentiable. Let $x^* \in \mathbb{R}^n$ be a local minimum of f on every straight line through x^*, i.e., the functions

$$g_d(t) = f(x^* + td)$$

all have a local minimum at $t = 0$ for all $d \in \mathbb{R}^n$.

a) Show that $\nabla f(x^*) = 0$.

b) Let $\tilde{x}$ be a local minimum of f. Show that $\tilde{x}$ is a local mininum of f on every straight line through $\tilde{x}$.

c) Let $f(x_1, x_2) = (x_2 - px_1^2)(x_2 - qx_1^2)$ with $0 < p < q$. Show that $x^* = (0,0)^\top$ is a local minimum of f on every straight line through x^*. Also show that x^* is not a local minimum of f.

(6 points)

Exercise 3. (gradient descent)

We consider the gradient descent method with a constant stepsize $\sigma > 0$.

a) Let $f : \mathbb{R}^n \to \mathbb{R}$ be given as $f(x) = \|x\|_2^{3/2}$. Show that ∇f is not Lipschitz-continuous on $\mathbb{R}^n \setminus \{0\}$. Furthermore, show that the gradient descent method with constant stepsize either reaches the gobal minimum $x^* = 0$ after a finite number of steps or does not converge to x^* at all.

b) Let $f : \mathbb{R}^n \to \mathbb{R}$ be given as $f(x) = \|x\|_2^{2+\beta}$ with $\beta > 0$. For which x_0, σ does the gradient descent method converge/diverge?

(4 points)