Scientific Computing II

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Sheet 0

Submission on -.

Exercise 1. (weak differentiability)

We consider the function $f \in L^1_{\text{loc}}(\mathbb{R})$, $f(x) = |x|$.

a) Show that f is weakly differentiable and compute its weak derivative.

b) Show that f is not twice weakly differentiable.

(0 points)

Exercise 2. (weak formulation I)

Let $\Omega \subset \mathbb{R}^d$ be open and bounded, with smooth boundary $\partial\Omega$. Let $n(x)$ be the outer normal vector. For functions $u \in \mathcal{C}^2(\Omega)$, we consider the PDE

$$\begin{aligned} -\Delta u(x) &= f(x) \text{ in } \Omega \\ u(x) &= 0 \text{ on } \partial\Omega \end{aligned}$$

Derive the corresponding weak formulation in the space

$$H^1_0(\Omega) = \{u \in H^1(\Omega) \mid u(x) = 0 \text{ on } \partial\Omega\}\,.$$

(0 points)

Exercise 3. (higher regularity in 1D)

Let $I = [a, b] \subset \mathbb{R}$ and $f \in L^2(I)$. Let $u \in H^1_0(I)$ be the weak solution to the Poisson equation

$$-u'' = f$$

with Dirichlet boundary conditions. Show that u belongs to $H^2(I)$.

(0 points)

Exercise 4. (weak formulation II)

Let $\Omega \subset \mathbb{R}^d$ be open and bounded, with smooth boundary $\partial\Omega$. Let $n(x)$ be the outer normal vector. For functions $u \in \mathcal{C}^4(\Omega)$, we consider the PDE

$$\begin{aligned} \Delta[\Delta u](x) &= f(x) \text{ in } \Omega \\ u(x) &= 0 \text{ on } \partial\Omega \\ \nabla u(x) \cdot n(x) &= 0 \text{ on } \partial\Omega\,. \end{aligned}$$

Derive the corresponding weak formulation in the space

$$H^2_0(\Omega) = \{u \in H^2(\Omega) \mid u(x) = 0 \text{ on } \partial\Omega,\ \nabla u(x) \cdot n(x) = 0 \text{ on } \partial\Omega\}\,.$$

(0 points)