# Scientific Computing II

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## Sheet 3

Submission on **Tuesday, 8.5.18**.

**Exercise 1.** (weak-* convergence)

Let $Y = (0,1)^n$ and $a \in L^\infty_\sharp(Y)$. For $a^\epsilon(x) = a(x/\epsilon)$, show that

$$a^\epsilon \rightharpoonup \int_Y a(y)\,\mathrm{d}y$$

weakly-* in $(L^1(\mathbb{R}^n))'$.

(4 points)

**Exercise 2.** (two-scales convergence)

a) Let $Y = (0,1)^n$ and $a \in C^0(\Omega \times Y)$ with $a(x,\cdot)$ $Y$-periodic and $a_\epsilon(x) = a(x, x/\epsilon)$. Then one has

$$a^\epsilon \rightharpoonup a$$

in the two-scales sense and

$$a^\epsilon \rightharpoonup \int_Y a(\cdot, y)\,\mathrm{d}y$$

weakly in $L^2(\Omega)$.

b) If $u^\epsilon \to u$ strongly in $L^2(\Omega)$, then also $u^\epsilon \rightharpoonup u$ in the two-scales sense.

c) If $u^\epsilon$ has an asymptotic expansion

$$u^\epsilon(x) = \sum_{i \in \mathbb{N}} \epsilon^i u_i\left(x, \frac{x}{\epsilon}\right)$$

with $Y$-periodic functions $u_i \in C^0(\Omega \times Y)$, one has $u^\epsilon \rightharpoonup u_0$ in the two-scales sense.

(6 points)