Scientific Computing II

Summer term 2018
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Sheet 6

Submission on **Thursday, 21.6.18**.

Exercise 1. (1D heat equation)

We again consider Exercise 1 from the previous sheet. There, we used the implicit Euler scheme and a finite element approach to arrive at the discrete system (1). A generalization of the time discretization can be obtained with the θ-rule: For $\theta \in [0, 1]$, one approximates

$$\int_a^b g(x)\,\mathrm{d}x \approx (b-a)(\theta g(a) + (1-\theta)g(b))\,.$$

Using this approach instead of implicit Euler, derive the more general time-space discretized formulation

$$\left(M + (1-\theta)\frac{T}{N}K\right)\underline{y}^n = \theta\frac{T}{N}L^{n-1} + (1-\theta)\frac{T}{N}L^n + \left(M - \theta\frac{T}{N}K\right)\underline{y}^{n-1}\,, \quad n = 1, \ldots, N\,.$$

(6 points)

Programmieraufgabe 1. (1D heat equation)

- Modify your code from the last exercise sheet to solve the θ-rule based system of equations instead. Try to solve the heat equation from the previous sheet with $T = 1$, $f \equiv 0$, $l \equiv r \equiv 0$, and

  $$y(x,0) = \begin{cases} 5 & x \leq 0.5 \\ 0 & \text{else.} \end{cases}$$

  For $\theta = 0, 0.5, 1$, run your program for different choices of m, N.

- For certain choices of m, N, one can see 'ripples' in the solution which get smoothened out rather slowly. Try to reproduce this behaviour with your program. Does this have an effect on L^2/H^1-convergence?

(14 points)

The programming exercise should be handed in either before/after the exercise class on 21.6.18 (bring your own laptop!) or in the HRZ-CIP-Pool, after making an appointment at 'angelina.steffens@uni-bonn.de'. All group members need to attend the presentation of your solution. Closing date for the programming exercise is the 21.6.2018. You can choose the programming language yourself.