# Scientific Computing II

Sommer Semester 2019
Prof. Dr. Carsten Burstedde
Biagio Paparella

## Exercise Sheet 1.

Due date: **Tue, 16.04.2019**.

**Exercise 1.** (Compactly supported functions form a proper subspace) (5 Points)

Let $\Omega$ be a bounded domain. With the help of Friedrichs's inequality, show that the constant function $u = 1$ is not contained in $H_0^1(\Omega)$, and thus $H_0^1(\Omega)$ is a proper subspace of $H^1(\Omega)$.

**Exercise 2.** (PF inequality for higher order) (5 Points)

Prove the following claim: If $\Omega$ is bounded, then $|\cdot|_m$ is a norm on $H_0^m(\Omega)$ which is equivalent to $\|\cdot\|_m$. If $\Omega$ is contained in a cube with side length $s$, then $|v|_m \leq \|v\|_m \leq (1+s)^m |v|_m$ for all $v \in H_0^m(\Omega)$

*Hint: induction.*

**Exercise 3.** (Function space inclusions) (5 Points)

Propose a function in $C[0,1]$ which is not contained in $H^1[0,1]$. To illustrate that $H_0^0(\Omega) = H^0(\Omega)$, propose a sequence in $C_0^\infty(\Omega)$ which converges to the constant function $v = 1$ in the $L_2[0,1]$ sense.

**Exercise 4.** (Directional derivatives) (5 Points)

By using the definitions given in class, compute the directional derivatives of:

- $f : U \times V \to U$, given by $(u, v) \mapsto u$,
- $g : U \to U \times V$, given by $u \mapsto (u, c)$, for a fixed constant $c \in V$,
- $h : U \times V \to W$, $(u, v) \mapsto h(u, v)$ bilinear.

The chain rule reads: $D_x(f \circ g)(\bar{x}) = D_{g(x)} f \circ D_x g(\bar{x})$ .
By using exclusively the rules above, prove the *product* rule for real-values functions: Given $f, g : U \to \mathbb{R}$, show that $D_x(fg)(\bar{x}) = D_x f(\bar{x}) g(x) + f(x) D_x g(\bar{x})$

*P.S.: in this exercise it is not required to show full Fréchet differentiability.*