Scientific Computing II

Sommersemester 2019

Prof. Dr. Carsten Burstedde
Biagio Paparella

Exercise Sheet 10.

Due date: **25.06.2019**.

Exercise 1. (Adjoint operator: Hilbert space case) (5 Points)

Let $L : X \to Y$ be a linear operator between Banach spaces. We defined its adjoint to be the map $L' : Y' \to X'$ behaving like $L'(l) = l(L)$ for $l \in Y'$.

On the other hand, for an operator $A : H \to K$ between Hilbert spaces, the definition for the adjoint A^* relies on the scalar product: it is the unique map $A^* : K \to H$ such that $(Ah, k)_K = (h, A^*k)_H$. Explain in details why the two definitions are perfectly compatible.

Exercise 2. (An easy density criteria) (5 Points)

Let H be a complex Hilbert space and $T : D(T) \to H$ a linear operator, where $D(T) \subset H$ is a dense subspace in H ($D(T) = H$, possibly). Prove that if $(u, Tu) = 0$ for any $u \in D(T)$ then $T = 0$, i.e. T is the null operator (sending everything to 0).

Exercise 3. (The solution u is continuous w.r.t f) (5 Points)

Let's work in the space $H = H_0^m(\Omega)$, where we try to solve an elliptic PDE with our usual notation (f for the right side, a the elliptic bilinear form with constant α, u the weak solution). Prove that:

(i) for each $v \in H$, $\|v\|_0^2 \le \|v\|_{-m}\|v\|_m$

(ii) $\|u\|_m \le \alpha^{-1}\|f\|_{-m}$

Exercise 4. (From a linear mapping to an elliptic form) (5 Points)

Let $a : V \times X \to \mathbb{R}$ be a positive symmetric bilinear form satisfying the hypothesis of theorem 2.7 (following our notes, or equivalently 3.6 from Braess). Show that a is elliptic, i.e. $a(v, v) \ge \alpha\|v\|_V^2$ for some $\alpha > 0$.