Scientific Computing II

Sommersemester 2019
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Exercise Sheet 2.

Due date: **Tue, 23.04.2019**.

Exercise 1. (An example for a non-closed range) (5 Points)

In connection with example 1.17, consider the linear mapping:

$$L : l_2 \to l_2$$

$$(Lx)_k = 2^{-k} x_k$$

Show that it's continuous and its range is not closed.
Hint: The closure contains the point $y \in l_2$ with $y_k = 2^{-k/2}$, $k = 1, 2, \ldots$

Exercise 2. (An ODE example, part 1 - corrected typo (H^1, not H_0^1)) (5 Points)

For functions $u : [0, 1] \to \mathbb{R}$, consider the problem:

$$-u'' + u' + u = f \quad on \quad (0, 1), \qquad u'(0) = u'(1) = 0$$

We choose the following variational formulation:

- $V = \{f \in H^1(0, 1) \text{ s.t. } f'(0) = f'(1) = 0\}$
- $a(u, v) = \int_0^1 (u'v' + u'v + uv)\mathrm{d}x$
- $F(v) = (f, v)$

Show that the form $a(\cdot, \cdot)$ is *not* symmetric, but is continuous and coercive.

Exercise 3. (An ODE example, part 2) (5 Points)

By referring to the previous exercise, what happens to coercivity if the equation is replaced by

$$-u'' + ku' + u = f$$

for a free parameter $k \in \mathbb{R}$?

Exercise 4. (Another ODE example) (5 Points)

Consider the symmetric, positive definite and continuous bilinear form on $H_0^1(0, 1)$:

$$a(u, v) = \int_0^1 x^2 u(x)' v(x)' \mathrm{d}x$$

Consider the functional $J : H \to \mathbb{R}$ defined as $J(v) \doteq \frac{1}{2} a(v, v) - \int_0^1 v \mathrm{d}x$. Find the appropriate operator L and $f \in H$ s.t. the weak solution of $Lu = f$ on $(0, 1)$, $u(0) = u(1) = 0$ is defined to be the minimizer of J. Anyway, some computations show that such an ODE does not admit solutions in H. How do you explain this phenomenon?