Scientific Computing II

Sommersemester 2019
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Exercise Sheet 5.

Due date: **14.05.2019**.

Theorem (Trace theorem)**.** Let Ω be bounded, and suppose Ω has a piecewise smooth boundary $\Gamma = \partial\Omega$. In addition, suppose Ω satisfies the cone condition[1]. Then there exists a bounded linear mapping:

$$\gamma : H^1(\Omega) \to L^2(\Gamma), \quad \|\gamma(v)\|_{0,\Gamma} \leq c\|v\|_{1,\Omega}$$

such that $\gamma v = v_{|\Gamma}$ for all $v \in C^1(\bar{\Omega})$ (the latter is the restriction of v on Γ).

Exercise 1. (The space H_0^1 already contains the zero boundary conditions) (5 Points)

Suppose that the domain Ω has a piecewise smooth boundary and satisfies the cone condition, and let $u \in H^1(\Omega) \cap C(\bar{\Omega})$. By using the trace theorem, show that $u \in H_0^1(\Omega)$ implies $u = 0$ on $\partial\Omega$ (the converse is also true, but you're not required to prove it).

Exercise 2. (A simple reformulation can lead to drastically different results)(5 Points)

Iteration methods are based on the Banach Fixed-Point Theorem. In few words, for finding an u s.t. $u = T(u)$, one starts with an initial guess u_0 and then iterates by $u_{n+1} = T(u_n)$. As an example, suppose we want to compute the positive square root of 2, i.e. the root of $x^2 - 2 = 0$.

- (a) The requested value satisfies $x = 2/x$. Prove that the derived iteration method does *not* converge unless $x_0 = \sqrt{2}$;
- (b) Note that $\sqrt{2} \in [1, 2]$ is also a fixed point for

  $$x = T(x) = \frac{1}{4}(2 - x^2) + x$$

  Prove that the derived iteration method converges for any $x_0 \in [1, 2]$.

Exercise 3. (Change of variable in J-GS) (5 Points)

Suppose that both the Jacobi and Gauss-Seidel methods converge for the equation $Ax = b$. Let D be a nonsingular diagonal matrix. Do we still get convergence if AD (or DA) is substituted for A?

Exercise 4. (A sufficient condition for a non-negative inverse) (5 Points)

A matrix B is called *non-negative*, written as $B \geq 0$, if all matrix elements are non-negative. Let D, L, $U \geq 0$ (recall: D is diagonal, L lower-triangular, U upper-triangular), and suppose that the Jacobi methods converges for $A = D - L - U$. Show that this implies $A^{-1} \geq 0$.

[1] briefly speaking, on each vertex a cone can be positioned to be inside Ω; there are no "cusps".