Scientific Computing II

Sommersemester 2019
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Exercise Sheet 8.

Due date: **04.06.2019**.

Programming Exercise 1. (Jacobi vs. SOR) (5 Points)

We will add a small and simple programming exercise for the effect. Let us choose the Poisson matrix with n rows and columns (for $n = 3$, it looks as follows),

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}. \tag{1}$$

For simplicity, fix $n = 3$ and consider the linear system $Ax = 0$. Write a C program for solving the problem with the use of both the Jacobi and SOR algorithms. For the latter, choose

$$\omega = \frac{2}{1 + \sin\left[\frac{\pi}{n+1}\right]}. \tag{2}$$

Explain intuitively why I proposed such a value, and compare the rate of convergence of the two methods. Please attach a plot "number of iterations vs. error."

Exercise 1. (A stronger convergence rate) (Points)

When using the s-norm $|||\cdot|||_s$, it is possible to derive a recursive formula for the convergence rate of a W-cycle

$$\rho_l \leq \rho_1 + \rho_{l-1}^2(1 + \rho_1) \tag{3}$$

Show that by using instead the energy norm[1] this rate can be improved to

$$\rho_l^2 \leq \rho_1^2 + \rho_{l-1}^4(1 - \rho_1^2) \tag{4}$$

Hint: proceed by induction on levels l; use the error-orthogonality w.r.t. a

Exercise 2. (A good bound for the first level is enough) (5 Points)

Prove that when $\rho_1 < \sqrt{1/2}$ a W-cycle admits the convergence rate

$$\sup_l \rho_l^2 \leq \frac{\rho_1^2}{1 - \rho_1^2} \tag{5}$$

Exercise 3. (The nonlinear algorithm extends the previous case) (5 Points)

Verify that the nonlinear multigrid algorithm 2.34 reduces to the known multigrid method for linear problems.

[1] the norm induced by the bilinear map $a(\cdot,\cdot)$ on S_h (def. 1.3.9b)