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Computer lab Numerical Algorithms
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Problem sheet 3 **November 14th, 2012**

So far we considered the cell problem separately, i.e. for a given linear profile u_H we computed the periodic correction u_h with $\fint u_h \, \mathrm{d}x = 0$ by solving

$$\int_Q a^\epsilon \, \nabla\big(u_h + u_H\big) \cdot \nabla \varphi_h \, \mathrm{d}x = 0 \quad \forall\, \varphi_h \in \mathcal{V}_{h,\#} \,.$$

Now we want to tackle the macroscopic problem: Find u_H such that

$$a_H(u_H, \varphi_H) = (f\,, \varphi_H) \quad \forall\, \varphi_H \in \mathcal{V}_H$$

where $f \in L^2$ and

$$a_H(\varphi_H, \psi_H) := \sum_{T \in \mathcal{T}_H} \sum_{\gamma_T \in T} \omega_{\gamma_T} \underbrace{\fint_{\epsilon Q} a^\epsilon_{\gamma_T}(x)\, \nabla R_h(\varphi_H)(x) \cdot \nabla R_h(\psi_H)(x) \, \mathrm{d}x}_{=(a^{H,h}\,\nabla\varphi_H \cdot \nabla\psi_H)(\gamma_T)} \,.$$

Furthermore, $R_h(\varphi_H)(x) \in \varphi_H(\gamma_T) + \nabla\varphi_H(\gamma_T)(x - \gamma_T) + \mathcal{V}_{h,\#}(\gamma_T)$ is given as the solution of the cell problem, i.e.

$$\int_{\epsilon Q} a^\epsilon_{\gamma_T}(x)\, \nabla R_h(\varphi_H)(x) \cdot \nabla \varphi_h(x) \, \mathrm{d}x = 0 \quad \forall\, \varphi_h \in \mathcal{V}_{h,\#}(\gamma_T) \,.$$

That means we have to solve the cell problem for every quadrature point γ_T. Hence it is convenient to introduce a class CellProblem that provides a function CellProblem ::evaluateMicroEnergy which evaluates $(a^{H,h}\,\nabla\varphi_H \cdot \nabla\psi_H)(\gamma_T)$. This evaluation should be done by first computing the reconstruction operators $R(\varphi_H)$ and $R(\psi_H)$ and then applying a (weighted) stiffness operator to evaluate the integral $(a^{H,h}\,\nabla\varphi_H \cdot \nabla\psi_H)(\gamma_T)$.

To avoid assembling and solving the cell problem numerous times we can make use of the linearity of the problem. Recall, we assume that the macroscopic solution is given by a linear profile plus a periodic correction. Hence we introduce so called *base solutions* b_i as members of the class CellProblem which are computed once when constructing the class. The base solutions are obtained by creating linear profiles in the direction of the unit basis vectors e_i to set up the right hand side of the usual cell problem and then solve the PDE as usual. A general solution for $R(\varphi_H)$ is then obtained by passing $\nabla\varphi_H(\gamma_T)$ and computing $R(\varphi_H) = \sum_i \partial_i \varphi_H(\gamma_T) b_i$.

Tasks:

- Complete the class CellProblem, i.e. fill CellProblem::recomputeBaseSolution() and CellProblem::evaluateMicroEnergy().
- Write a main program to test this class! This can be done by computing and comparing two solutions for the same problem: the first by using the methods described on earlier lab sheets and the second by using the base solutions computed in the class CellProblem and multiplying them with the right coefficients.