Institut für Numerische Simulation
Rheinische Friedrich-Wilhelms-Universität Bonn

universitätbonn

Tutorial Numerical Algortihms

Winter term 2012/2013

Prof. Dr. M. Rumpf – B. Geihe, B. Heeren

Problem sheet 1 — October 12th, 2012

Problem 1 (Constrained optimization in $\mathbb{R}^d$)

Consider some differentiable functions $F : \mathbb{R}^d \to \mathbb{R}$ and $G : \mathbb{R}^d \to \mathbb{R}^k$ with $0 < k < d$, such that

$$\mathcal{M} := [G = 0] := \{x \in \mathbb{R}^d : G(x) = 0\}$$

is a differentiable manifold of dimension $d - k$.

Write down the Langrangian for the constrained optimization problem

$$\min_{x \in \mathcal{M}} F(x)$$

and derive a Newton method to solve this problem.

Problem 2 (Discretization of a cell problem)

We consider the weak formulation of a microscopic cell problem as posed in the lecture: For a given function u_H we want to compute a microscopic profile u_h such that

$$\int_Q a(x) \nabla u_h \cdot \nabla \varphi_h \,\mathrm{d}x = - \int_Q a(x) \nabla u_H \cdot \nabla \varphi_h \,\mathrm{d}x \quad \forall\, \varphi_h \in \mathcal{V}_h$$

where $\mathcal{V}_h$ is the usual finite element space of piecewise linear functions defined on a triangulation of $Q = [0, 1]^2$.

(a) Discretize the equation by using the corresponding basis $\{\varphi_i\}_i$ of $\mathcal{V}_h$ and numerical quadrature. Derive the resulting linear system of equations $A\,\bar{u}_h = \bar{b}$.

(b) Does this problem possess a unique solution? Which prerequisites are needed to derive this weak formulation from $-\mathrm{div}\,(a(x)(u_H + u_h)) = 0$?

Problem 3 (Handling periodic boundary conditions)

We want to include periodic boundary conditions for the equation of problem 2, i.e. we want to indentify degrees of freedom lying on $\{x = 0\}$ and $\{y = 0\}$ with the corresponding ones on $\{x = 1\}$ and $\{y = 1\}$ respectively. Given the linear system of equations $A\,\bar{u}_h = \bar{b}$ derive a modified version which incorporates these periodic boundary conditions.

Problem 4 (Lab exercise)

See additional sheet.