

Tutorial Numerical Algorithms

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Problem sheet 4

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Problem 1 (Effective tensor in 1D)

Let $Q = [0, 1]$ and $a \in L^\infty(Q)$ with $a(x) > c > 0$. Compute the effective tensor a^* .

Problem 2 (Effective tensor for a rotated bar pattern in 2D)

Consider $Q = [0, 1]^2$ and $a \in L^\infty(Q)$ with

$$a(y_1, y_2) = \begin{cases} a_1, & 0 \leq (y_1 - y_2) - \lfloor y_1 - y_2 \rfloor \leq \lambda \\ a_2, & \lambda < (y_1 - y_2) - \lfloor y_1 - y_2 \rfloor \leq 1 \end{cases}$$

for $a_{1,2} \in \mathbb{R}$ and $0 < \lambda < 1$. Compute the effective tensor a^* .

Problem 3 (Inverse inequality)

Show that for $\varphi_h \in V_h$ and \mathcal{T}_h with constant grid size

$$|\varphi_h|_{m,p,D} \leq Ch^{\tau-m} \|\varphi_h\|_{\tau,p,D}$$

for all integers $0 \leq \tau < m$.