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Tutorial Numerical Algorithms
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Problem sheet 6 **December 5th, 2012**

Problem 1

Under the usual assumptions (with $k = 1$) show that

$$\|u^\epsilon - u_H^\epsilon\|_{0,2,D} \leq C\,(\epsilon + H^2)\,.$$

Hint: Estimate $u_H^\epsilon - u^\epsilon = (u_H^\epsilon - \hat{R}^\epsilon(u_H)) + (\hat{R}^\epsilon(u_H) - u_H) + (u_H - u^*) + (u^* - u^\epsilon)$ by using Poincare for the first term, i.e. show first that $⨍_{Q_\epsilon} u_H^\epsilon - \hat{R}^\epsilon(u_H)\ \mathrm{d}x = 0$.

Problem 2 (Properties of the homogenized elasticity tensor)

Show that the homogenized elasticity tensor C^* fulfills the symmetry property $C^*_{ijkl} = C^*_{klij}$ and that

$$C^*(x)\,B : B = \inf_{\chi \in H^{1,2}_\#(Q,\mathbb{R}^d)} ⨍_Q C(x,y)\,(B + \varepsilon[\chi]) : (B + \varepsilon[\chi])\,\mathrm{d}y$$

for symmetric matrices B.

Problem 3 (Discretization of the elasticity problem)

For $u : D \to \mathbb{R}^d$ consider the elastic energy

$$E[u] = \frac{1}{2}\int_D C\varepsilon[u] : \varepsilon[u] - f \cdot u\,\mathrm{d}x\,,$$

where $f \in L^2(D,\mathbb{R}^d)$, $u = u^\partial$ on ∂D, $C = C_{ijkl}$ and $\varepsilon_{ij}[u] + \frac{1}{2}(u_{i,j} + u_{j,i})$. Set up a Finite Element Method for this vector valued problem and describe the structure of the stiffness matrix.

Problem 4 (Lamé-Navier material)

For $\mu, \lambda \geq 0$ consider the elastic energy

$$E[u] = \frac{1}{2}\int_D \lambda\,(\mathrm{tr}\,\varepsilon[u])^2 + 2\mu\,\varepsilon[u] : \varepsilon[u]\,\mathrm{d}x\,.$$

Derive the elasticity tensor C and describe the resulting modifications of the stiffness matrix.