Institut für Numerische Simulation
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Tutorial Numerical Algorithms

Winter term 2012/2013

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Problem sheet 7 **December 11th, 2012**

Problem 1 (Isometric deformations)

Show that an isometric deformation $\phi \in C^2(\mathbb{R}^d, \mathbb{R}^d)$ is a rigid body motion, i.e.

$$\|\phi(x) - \phi(y)\| = \|x - y\| \ \forall x, y \quad \Leftrightarrow \quad \phi = Qx + b,\ Q \in SO(d),\ b \in \mathbb{R}^d .$$

Hint: Compute $\partial_x \partial_y \left(\|\phi(x) - \phi(y)\|^2 - \|x - y\|^2 \right)$.

Problem 2 (Korn's first inequality)

For a piecewise smooth domain D and $u \in H^{2,2}_{\partial D}(D)$ show the following part of Korn's first inequality:

$$\|\mathrm{D}u\|_{0,2,D} \leq \sqrt{2}\, \|\varepsilon[u]\|_{0,2,D} .$$

Hint: Apply partial integration to $\int_D \epsilon[u]^2 \,\mathrm{d}x$.

Problem 3 (Symmetry of the stress tensor)

Prove that the axiom of moment balance implies symmetry of the stress tensor T^ϕ, i.e.

$$\int_{V^\phi} x^\phi \times f^\phi(x^\phi) \mathrm{d}x^\phi + \int_{\partial V^\phi} x^\phi \times t^\phi(x^\phi, n^\phi) \mathrm{d}a^\phi = 0 \ \ \forall\, V^\phi \subset D^\phi \ \Rightarrow \ T^\phi(x^\phi) = T^\phi(x^\phi)^\top .$$

Hint: Use Gauss's Theorem to transform the surface integral.