Numerical Algorithms

Winter semester 2013/2014
Prof. Dr. Carsten Burstedde
Philipp Morgenstern

universität**bonn**

Exercise Sheet 1.

Due date: **Thursday, 31 October**.

Exercise 1. (PDE under transformation)

Consider the equation $Lu = f$ with a second order linear partial differential operator L in some open and connected domain $\Omega \subset \mathbb{R}^d$, and a transformation $\Phi : \Omega \to \Omega' \subset \mathbb{R}^d$ with continuous, differentiable and non-singular derivative $D\Phi := \frac{\partial \Phi}{\partial x}$.

Prove that the equation does not change its type when formulated in terms of transformed coordinates $\xi = \Phi(x)$.

(4 points)

Exercise 2. (weak derivatives)

a) If u has a weak derivative $D^\alpha u$ in Ω, then u is also weakly differentiable in every connected subset $\Omega_0 \subset \Omega$, having the same derivative.

b) If $D^\alpha u$ has a weak derivative $D^\beta(D^\alpha u)$, then $D^{\alpha+\beta}u$ exists with $D^\beta(D^\alpha u) = D^{\alpha+\beta}u = D^\alpha(D^\beta u)$.

(2 points)