Numerical Algorithms

Winter semester 2013/2014
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Exercise Sheet 3.

Due date: **Tuesday, 12 November**.

Exercise 5. (function folding)

Show that the folding of L^2-functions

a) is distributive and

b) satisfies

$$\chi f * g = f * \chi g = \chi(f * g)$$

for any scaled characteristic function $\chi(x) = \begin{cases} a & x \in A \subseteq \mathbb{R} \\ 0 & \text{else} \end{cases}$.

If this is not true, give a counterexample.

(3 points)

Exercise 6. (Riesz representation)

Given $n \in \mathbb{N} \ni k \leq n$, find the coefficients $(g_0, \ldots, g_n) \in \mathbb{R}^{n+1}$ of $g \in P_n([0,1])$ such that

$$\int_0^1 f g \,\mathrm{d}x = \int_0^1 \frac{\partial^k}{\partial x^k} f \,\mathrm{d}x \quad \text{for all } f \in P_n([0,1]) \,.$$

Hint: It suffices to denote the inverse of the $(n+1)$-dimensional Hilbert matrix by H_{n+1}^{-1}.

(4 points)