Numerical Algorithms

Winter semester 2013/2014
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Exercise Sheet 8.

Due date: **Thursday, 23 January**.

Programming Exercise 3. (FE solver for parabolic PDE)

Use your existing program that solves an elliptic PDE as a starting point to solve the heat equation for $t \in [0, T]$,

$$u_t - \Delta u = 0 \qquad \text{in the unit square,} \tag{1a}$$
$$u = g(x,t) \qquad \text{on its boundary,} \tag{1b}$$
$$u = g(x,0) \qquad \text{at time zero.} \tag{1c}$$

Use the fundamental solution $\Phi(x,t)$ to define the exact solution, initial and boundary conditions,

$$g(x,t) = \Phi(x - x_0, t + t_0). \tag{2}$$

Place x_0 inside the unit square and pick t_0 and T such that the half width $\sigma = \frac{1}{5}$ for $g(x,0)$, and that $g(x,T) \leq 2$.
Implement the θ one-step method. Plot the L^2-error over time for different mesh spacings h, and plot the final-time error at $t = T$ over h.
Try this both for $\theta = 0$, where you make the time step bigger as long as the method is stable. Then switch to $\theta = \frac{1}{2}$ and experiment with even larger time step sizes.

(10 points)

Submit your solutions via email to morgenstern@ins.uni-bonn.de .
Deadline is the tutorial on Thursday, 23 January.