Scientific Computing I

Winter Semester 2013 / 2014
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Exercise sheet 13.

Closing date **28.1.2014**.

Theoretical exercise 1. (Estimate for the θ-scheme [5 points])

Assume $g \in C^1([0,T],\mathbb{R})$ and show that

$$\left| \int_0^1 g(t) - [(1-\theta)g(0) + \theta g(1)] \right| \leq \max(\theta, 1-\theta) \int_0^1 |g'(t)| \, \mathrm{d}t.$$

Theoretical exercise 2. (Parabolic initial value problem [5 bonus points])

Consider the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}(u(t),v)_H + a(u(t),v) = \langle f(t), v \rangle \quad \forall v \in V$$

for almost all $t \in (0,T)$ and the initial condition $u(0) = u_0$. Let us assume that $f \in C([0,T],V)$ and $u \in C^1([0,T],V)$, and note that

$$(u'(t),u(t))_H = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(u(t),u(t))_H \quad \forall t \in (0,T)\ .$$

Show that

$$\|u(t)\|_H \leq e^{-\kappa t}\|u_0\|_H + \int_0^t e^{-\kappa(t-s)}\|f(s)\|_H \mathrm{d}s \quad \forall t \in (0,T)$$

with $\kappa = \alpha/c^2$, where $\alpha > 0$ is the coercivity constant of $a(\cdot,\cdot)$ and c satisfies $\|v\|_H \leq c\|v\|_V$ for all $v \in V$.

Theoretical exercise 3. (Error estimate [5 bonus points])

Show that

$$\|\tilde{u}_j - u_j\| \leq e^{t_j L}\left[\|\psi_0(\tilde{u}_\tau)\| + \tau_0\|\psi_1(\tilde{u}_\tau)\| + \cdots + \tau_{j-1}\|\psi_j(\tilde{u}_\tau)\|\right]$$

for all $\tilde{u}_\tau \in X_\tau$ and $j = 0, 1, \ldots, m$. Particularly, for $j = m$ it holds that

$$\|\tilde{u}_\tau - u_\tau\|_{X_\tau} \leq e^{TL}\|\psi_\tau(\tilde{u}_\tau)\|_{Y_\tau}$$

for all $\tilde{u}_\tau \in X_\tau$. Check the Script "Introduction to Computational Mathematics" or the book "Numerische Mathematik" by W. Zulehner for the used notation.