Scientific Computing I

Winter Semester 2013 / 2014
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Exercise sheet 9.

Closing date **17.12.2013**.

Theoretical exercise 1. (Isoparametric finite elements [5 points])

Let $\hat{K} := \{(\hat{x}_1, \hat{x}_2) \in \mathbb{R}^2 : 0 \leq \hat{x}_1, \hat{x}_2 \leq 1\}$ be the reference element with vertices $\{(0,0), (1,0), (0,1), (1,1)\}$ and nodal basis functions from the bilinear polynomial space $\hat{P} := \text{span}\{1, \hat{x}_1, \hat{x}_2, \hat{x}_1\hat{x}_2\}$. Under what conditions on the isoparametric element τ_s is the mapping $\Phi_s : \hat{K} \to \tau_s$

a) affine-linear?

b) bijective?

Theoretical exercise 2. (Nodal basis functions [5 points])

Determine all nodal basis functions on the following reference elements:

a) Serendepity class
$\hat{K} := \{(\hat{x}_1, \hat{x}_2) \in \mathbb{R}^2 : 0 \leq \hat{x}_1, \hat{x}_2 \leq 1\}$ with vertices

$$\{(0,0), (\tfrac{1}{2}, 0), (1,0), (1, \tfrac{1}{2}), (1,1), (\tfrac{1}{2}, 1), (0,1), (0, \tfrac{1}{2})\}$$

and the polynomial space $\hat{P} := \text{span}\{1, \hat{x}_1, \hat{x}_2, \hat{x}_1\hat{x}_2, \hat{x}_1^2, \hat{x}_2^2, \hat{x}_1^2\hat{x}_2, \hat{x}_1\hat{x}_2^2\}$.

b) Tetrahedron
$\hat{K} := \{(\hat{x}_1, \hat{x}_2, \hat{x}_3) \in \mathbb{R}^3 : \hat{x}_1, \hat{x}_2, \hat{x}_3 \geq 0, \sum_{i=1}^3 \hat{x}_i \leq 1\}$ with vertices

$$\{(0,0,0), (1,0,0), (0,1,0), (0,0,1)\}$$

and the polynomial space $\hat{P} := \text{span}\{1, \hat{x}_1, \hat{x}_2, \hat{x}_3\}$.

c) Hexahedron
$\hat{K} := \{(\hat{x}_1, \hat{x}_2, \hat{x}_3) \in \mathbb{R}^3 : 0 \leq \hat{x}_1, \hat{x}_2, \hat{x}_3 \leq 1\}$ with vertices

$$\{(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$$

and the polynomial space $\hat{P} := \text{span}\{1, \hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_1\hat{x}_2, \hat{x}_1\hat{x}_3, \hat{x}_2\hat{x}_3, \hat{x}_1\hat{x}_2\hat{x}_3\}$.

Theoretical exercise 3. (Higher order triangle elements [5 points])

Let $t \geq 0$. In a triangle T, there are $s = 1 + 2 + \cdots + (t+1)$ vertices $z_1, \ldots, z_s$ aligned on lines according to Fig. 1. Prove that for every continuous function f on T, there exists exactly one polynomial p of degree t with

$$p(z_i) = f(z_i) \qquad \text{for} \quad i = 1, 2, \ldots, s\,.$$

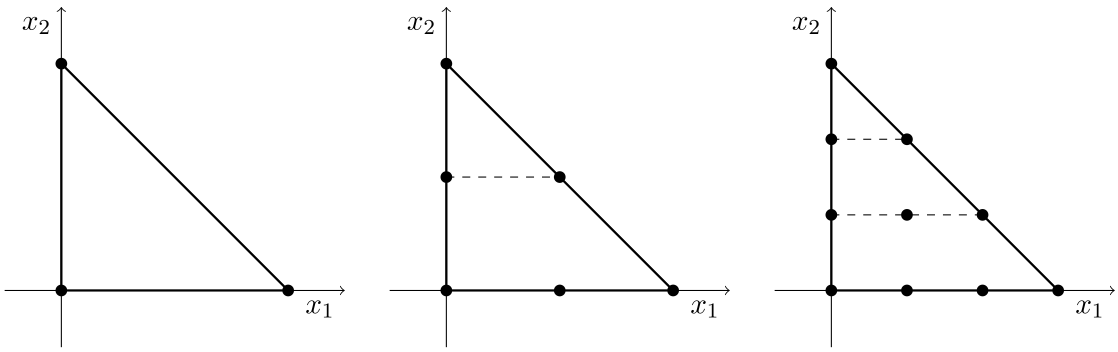

Figure 1: Vertices of the nodal basis for linear, quadratic and cubic triangle elements