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Exercise Sheet 1.

Due date: **Tuesday, 10.11.15**.

Exercise 1. (Traffic flow)

Consider the traffic flow of vehicles on a infinitely long highway. Let $\rho(x,t)$ denote the density of cars at position x and time $t > 0$. Assume that

$$0 \leq \rho \leq 1, \tag{1}$$

and that the velocity of cars v depends only on their density, $v = v(\rho)$. Furthermore, we take

$$v(\rho) = v_{max}(1-\rho), \quad \text{for some constant } v_{\max} > 0. \tag{2}$$

If the total number of cars is the conserved quantity, derive a PDE model for the traffic flow. What is the physical interpretation of conditions (1) and (2) ?

(4 points)

Exercise 2. (Burgers' equation)

Consider the initial value problem

$$\begin{cases} u_t + uu_x &= 0, \quad t > 0,\ x \in \mathbb{R}, \\ u(x,0) &= g(x), \quad x \in \mathbb{R}. \end{cases} \tag{3}$$

a) Prove that if the implicit equation

$$x = y + g(y)t \tag{4}$$

in the unknown y has a unique solution $y(x,t)$, then

$$u(x,t) = g(y(x,t)) \tag{5}$$

b) Let $g \in \mathcal{C}^1(\mathbb{R})$. Prove that the solution $u(x,t)$ of (3) also belongs to $\mathcal{C}^1(\mathbb{R})$ for all $(t,x) \in [0,\infty) \times \mathbb{R}$ if and only if $g'(x) \geq 0$ for all $x \in \mathbb{R}$.

(8 points)