Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 3.

Due date: **Tuesday, 24.11.15.**

Exercise 1. (Weak solutions)

For the initial value problem

$$\begin{cases} u_t + uu_x &= 0, \quad t > 0,\ x \in \mathbb{R}, \\ u(x,0) &= u_0(x), \quad x \in \mathbb{R}, \end{cases} \qquad u_0(x) = \begin{cases} 0 & \text{if} \quad x \leq 0, \\ 1 & \text{if} \quad x > 0. \end{cases} \tag{1}$$

Verify that the following functions:

$$u_1(x,t) = \begin{cases} 0 & \text{if} \quad x \leq \frac{t}{2}, \\ 1 & \text{if} \quad x > \frac{t}{2}, \end{cases} \qquad u_2(x,t) = \begin{cases} 0 & \text{if} \quad x \leq 0, \\ \frac{x}{t} & \text{if} \quad 0 < x \leq t, \\ 1 & \text{if} \quad t < x. \end{cases} \tag{2}$$

are both weak solutions.

(4 points)

Exercise 2. (Rankine-Hugoniot condition)

Show that the function $u_2(x,t)$ in (2) satisfies the Rankine-Hugoniot condition.

(3 points)

Exercise 3. (Breaking Time)

Consider the general scalar conservation law

$$\begin{cases} u_t + [f(u)]_x &= 0, \quad t > 0,\ x \in \mathbb{R}, \\ u(x,0) &= u_0(x), \quad x \in \mathbb{R}, \end{cases} \tag{3}$$

where $f \in \mathcal{C}^2(\mathbb{R})$, $|f''(y)| \leq M$ for some $M > 0$ and $u_0 \in \mathcal{C}^1(\mathbb{R})$. Find the minimum time t_B such that the solution of (3) fails to be in $\mathcal{C}^1(\mathbb{R})$.

(4 points)