Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 4.

Due date: **Tuesday, 1.12.15.**

Programming Exercise 1. (Godunov's method for linear systems)

Write a program `Godunov_linear_solv(A,q_l,q_r)` that given any $m \times m$ matrix A and states q_l and q_r approximates the solution of the Riemann problem:

$$\begin{cases} q_t + Aq_x = 0, & t > 0, x \in \mathbb{R} \\ q(x,0) = q_0(x), & t = 0. \end{cases} \qquad q_0(x) := \begin{cases} q_\mathrm{l}, & x < 0, \\ q_\mathrm{r}, & x > 0. \end{cases} \tag{1}$$

with the Godunov's method (see equation (1.4.32)) for $x \in [-2, 2]$ and $t \in [0, 4]$. The user should be able to choose the step sizes Δx and Δt for the corresponding discretization of these intervals. Additionally, the code must produce plots of the components of the approximate solution as a function of x for at least 3 distinct values of t or, optionally you can adapt the sample code to show an animation of one of the components of the solution for all the discrete times. Test out your program with

a) A scalar advection equation with $A = \bar{u} = 2$ and states $q_l = 0,\ q_r = 1$.

b) $A = \begin{pmatrix} 2 & 1 \\ 10^{-4} & 2 \end{pmatrix}$, $q_\mathrm{l} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $q_\mathrm{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

c) $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$, $q_l = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $q_r = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

Run your solver with different magnitudes of the ratio $c := \Delta t/\Delta x$. Report your findings when $c < 1$ and $c \geq 1$.

Update: Use periodic boundary conditions i.e $q(-2,t) = q(2,t)$ for all t.

(6 points)