Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 5.

Due date: **Tuesday, 8.12.15**.

Exercise 1. Prove Theorem 1.26.

(4 points)

Exercise 2. (Unit CFL condition)

Consider the advection equation with $\bar{u} > 0$,

$$\begin{cases} q_t + \bar{u} q_x &= 0, \quad t > 0, \\ q(x,0) &= q_0(x). \end{cases} \tag{1}$$

Application of the upwind method to this equation with a time step such that $\bar{u}\Delta t = \Delta x$, leads to the scheme

$$Q_i^{n+1} = Q_{i-1}^n. \tag{2}$$

That is, the initial data shifts one grid cell each time step and the exact solution is obtained up to accuracy of the initial data. In particular, if Q_i^0 is the exact cell average of $q_0(x)$, then the numerical solution will be the exact cell average for every time step. This property is called the *Unit CFL condition*.

a) Does the Lax-Friedrichs method satisfy the unit CFL condition. ?

b) Show that the exact solution (in the sense described above) is also obtained for the constant-coefficient acoustic equations (1.3.16) with $\bar{u} = 0$ if we apply Godunov's method with a time step such that $\bar{c}\Delta t = \Delta x$, where $\bar{c}$ is the speed of sound. Determine the formulas for p_i^{n+1} and u_i^{n+1} that result in this case.

c) Is it possible to obtain an exact result as in *b*) by a suitable choice of Δt in the case $\bar{u} \neq 0$.?

(6 points)