Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 7.

Due date: **Tuesday, 22.12.15.**

Exercise 1. (Godunov's flux function)

For the case of a convex flux f, show that formula (2.1.25) is equivalent to formula (2.1.26) by proving each of the statements of Property 2.4.

(3 points)

Exercise 2. (Godunov's flux for Burgers equation)

Verify that the numerical flux $\mathcal{F} = \dfrac{(Q^\star)^2}{2}$, where $Q^\star = Q^\star(Q_i, Q_j)$ is defined as follows:
If $Q_i \geq Q_j$ then

$$Q^\star := \begin{cases} Q_i, & \text{if } \dfrac{Q_i + Q_j}{2} > 0, \\ Q_j, & \text{else.} \end{cases} \tag{1}$$

If $Q_i < Q_j$ then

$$Q^\star := \begin{cases} Q_i, & \text{if } Q_i > 0, \\ Q_j, & \text{if } Q_j < 0, \\ 0, & \text{if } Q_i \leq 0 \leq Q_j. \end{cases} \tag{2}$$

corresponds to formula (2.1.26)

(2 points)

Exercise 3. (Harten's TVD test)

Prove that if $D_i^n, C_i^n \in \mathbb{R}$ are such that

$$C_{i-1}^n \geq 0, \quad \text{for all } i, \tag{3a}$$

$$D_i^n \geq 0, \quad \text{for all } i, \tag{3b}$$

$$C_i^n + D_i^n \leq 1, \quad \text{for all } i. \tag{3c}$$

Then a method of the form

$$Q_i^{n+1} = Q_i^n - C_{i-1}^n (Q_i^n - Q_{i-1}^n) + D_i^n (Q_{i+1}^n - Q_i^n) \tag{4}$$

is TVD.

(4 points)

Exercise 4. (Application of Harten's TVD test)

Show that if $\bar{u} < 0$ we can apply the Harten's test to the flux-limiter method (2.2.30) by choosing

$$C_{i-1}^n = 0, \tag{5a}$$

$$D_i^n = -\nu + \frac{1}{2}\nu(1+\nu)\left(\phi(\theta_{i+1/2}^n) - \frac{\phi(\theta_{i-1/2}^n)}{\theta_{i-1/2}^n}\right). \tag{5b}$$

in order to show that the method is TDV for $-1 \leq \nu \leq 0$ and that the bound (2.2.32) holds.

(4 points)