Numerical Algorithms

Winter Semester 2015
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Exercise Sheet 9.

Due date: **Tuesday, 12.1.16**.

Programming Exercise 1. (Jacobi polynomials)

a) Write a function `JacobiP(x, alpha, beta, N)` that uses the recursion (3.2.7) to evaluate the Jacobi polynomial $P_N^{(\alpha,\beta)}(x)$ at the points of a given vector x.

b) Write a `JacobiDer(x, alpha, beta, N)` that uses property (3.2.6) to compute $\frac{d}{dx}P_n^{(\alpha,\beta)}$ at the points of a given vector x.

c) Write a `JacobiQuad(alpha, beta, N)` that find the nodes and weights for the Gauß quadrature by finding the eigenvalues and the first eigenvector of the matrix T defined in (3.2.19).

d) Write a `JacobiLGL(alpha, beta, N)` that find the nodes and weights for the Gauß-Lobatto quadrature.

e) Write a function to obtain the Vandermonde $\mathcal{V}_N$ and Differentiation $\mathcal{D}_N$ matrices corresponding to the Legendre polynomial basis and Gauß-Lobato points of order N.

(10 points)

Programming Exercise 2. (Discrete derivatives)

a) Use the differentiation matrix $\mathcal{D}_N$ to compute the discrete derivative of

$$u(x) = \exp(\sin(\pi x)). \tag{1}$$

Present plots of the analytic derivative and the approximated one. Additionally plot the L_2–norm of the error for $N = 1, 2, \ldots, 64$.

b) Consider the sequence of functions defined by

$$u^{(0)}(x) = \begin{cases} -\cos(\pi x), & x \in [-1, 0), \\ \cos(\pi x), & x \in [0, 1], \end{cases} \qquad \frac{du^{(k+1)}}{dx} := u^{(k)}, \quad k \geq 0, \tag{2}$$

Repeat the part a) for $u^{(1)}(x)$ and $u^{(2)}(x)$.

(4 points)