Sheet 0.1

Submission on **-**.

Exercise 1. (parallelogram identity)

Let $(V, \|\cdot\|)$ be a Banach space over $\mathbb{R}$. Show that the following are equivalent:

1) there exists an inner product $(\cdot,\cdot)$ on V which induces $\|\cdot\|$, making V a Hilbert space
2) For all $u, v \in V$ one has $\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$

(0 points)

Exercise 2. (continuous functions and inner product)

Let $I = (-1, 1)$ and consider continuous functions $\mathcal{C}(I)$ and the inner product $(f, g) = \int_I f(x)g(x)\,\mathrm{d}x$. Show that $(\mathcal{C}(I), (\cdot,\cdot))$ is not a Hilbert space.

(0 points)

Exercise 3. (differentiable functions and inner product)

Let $I = (-1, 1)$ and consider continuously differentiable functions with homogeneous boundary conditions, i.e.

$$\mathcal{C}_0^1(I) = \left\{ f \in \mathcal{C}^1(I) \mid \lim_{x \to \pm 1} f(x) = 0 \right\}$$

and the inner product $(f, g) = \int_I f'(x)g'(x)\,\mathrm{d}x$. Show that $(\mathcal{C}_0^1(I), (\cdot,\cdot))$ is not a Hilbert space.

(0 points)