Scientific Computing 1

Winter term 2017/18
Priv.-Doz. Dr. Christian Rieger
Christopher Kacwin

Sheet 12

Submission on **Thursday, 25.1.18**.

Exercise 1. (Residual error)

Let $\Omega \subset \mathbb{R}^2$ be an open and bounded domain. We consider the problem:
Find $u \in H_0^1(\Omega)$ with

$$a(u, v) = F(v) \quad \forall v \in H_0^1(\Omega)\,,$$

where $a(u, v) = \int_\Omega \nabla u \nabla v \,\mathrm{d}x$ and $F(v) = \int_\Omega f v \,\mathrm{d}x$, $f \in L^2(\Omega)$. Furthermore, let $\mathcal{T}_h$ be a quasi-uniform triangulation of Ω and $\mathbb{V}_h \subset \mathbb{V}$ a conform finite element space. For the solution $u \in \mathbb{V}$ to the original problem and the approximation $u_h \in \mathbb{V}_h$ satisfying $a(u_h, v_h) = F(v_h)$ for all $v_h \in \mathbb{V}_h$, show that

$$a(u - u_h, v) = \sum_{T \in \mathcal{T}_h} \left(\int_T r_h v \,\mathrm{d}x + \frac{1}{2} \sum_{e \in \partial T \setminus \partial\Omega} \int_e R_h v \,\mathrm{d}S \right) \quad \forall v \in \mathbb{V}\,.$$

Here, $r_h = \Delta u_h + f$ denotes the residual on the corresponding element and $R_h = J(\nabla u_h \cdot n_e)$ denotes the jump of the normal derivative of u_h on the corresponding edge.

(4 points)

Exercise 2. (Trace theorem)

Consider an open and bounded domain $\Omega \subset \mathbb{R}^2$ with smooth boundary and the following version of the Trace theorem:

$$\|u\|_{0,\partial\Omega}^2 \le C_1 \|u\|_{0,\Omega}^2 + C_2 \|\nabla u\|_{0,\Omega}^2 \quad \forall u \in H^1(\Omega)\,.$$

Use a scaling argument to deduce the correct dependence of C_1, C_2 on $\mathrm{diam}(\Omega)$.

(4 points)