Scientific Computing 1

Winter term 2017/18
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Sheet 13

Submission on **Thursday, 1.2.18**.

Programmieraufgabe 1. (global system)

We finalize our finite element solver with this exercise. Recalling the last exercise sheet, we are able to generate the global system matrix in CSR format, as well as the global load vector. The final step is to solve the global system.

a) In the last exercise sheet, we generated a vector of nonzero global system matrix entries, from which we then constructed the global system matrix A. Write a routine that assembles the inverted diagonal of A

$$D^{\text{inv}}_{i,j} = \begin{cases} A^{-1}_{i,j} & i = j,\ A_{i,j} \neq 0\,, \\ 0 & \text{else}\,. \end{cases}$$

in CSR format.

b) Write a Matrix-Vector multiplication routine for CSR matrices that has runtime $\mathcal{O}(k)$ with k the number of nonzero entries in the matrix.

c) Implement the preconditioned conjugate gradient method (`https://en.wikipedia.org/wiki/Conjugate_gradient_method#The_preconditioned_conjugate_gradient_method`) which solves the system $Ax = b$ with the global system matrix A and preconditioning $M^{-1} = D^{\text{inv}}$. It is important to note here that rows/columns corresponding to boundary nodes are 0 in our matrices, the method therefore only makes sense for an initialization vector x_0 which also is 0 at boundary node indices.

d) Write a program that solves the PDE

$$\begin{aligned} -m\Delta u + cu &= f & \text{in } \Omega \\ u &= 0 & \text{on } \partial\Omega \end{aligned}$$

with $\Omega = [0,1]^2$, $m(x) = 1$, $c(x) = 0$, $f(x) = 1$. The neccesary steps are:

- Create a uniform triangular mesh with refinement parameter n
- Collect complete information about all elements and nodes
- Assemble the global load vector as well as the global system matrix and inverted diagonal in CSR format
- Solve the system with the preconditioned CG method, with initialization vector $x = 0$

e) The result is a vector $\bar{u}$ with a real value per node; the corresponding finite element solution $u_h = \sum \bar{u}_i \phi_i$ satisfies

$$u_h(x_i) = \bar{u}_i$$

for all nodes x_i. Visualize the solution for $n = 10, 50, 100, 500$, e.g. with gnuplot.

(16 points)

The programming exercise should be handed in during the exercise classes (bring your own laptop!) in one week, i.e. on 1.2/2.2.18. All group members need to attend the presentation of your solution to get points.