Scientific Computing 1

Winter term 2017/18
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Sheet 3

Submission on **Thursday, 9.11.17**.

Exercise 1. (ellipticity)

Consider the whole space $\Omega = \mathbb{R}^3$. Show that the following matrix functions $A\colon \Omega \longrightarrow \mathbb{R}^{3\times 3}$ are uniformly positive definite.

a) For every $x \in \Omega$, $A(x)$ is symmetric and has Eigenvalues $\lambda_1, \lambda_2, \lambda_3 > 0.1$.

b)

$$A(x) = \begin{bmatrix} 1.1 & \sin^2(x_1+x_2+x_3) & \cos^2(x_1+x_2+x_3) \\ \sin^2(x_1+x_2+x_3) & 2.1 & \sin(x_1+x_2+x_3) \\ \cos^2(x_1+x_2+x_3) & \sin(x_1+x_2+x_3) & 1.35 \end{bmatrix}$$

c)

$$A(x) = \begin{bmatrix} \sqrt{2}+0.1 & \sin(x_1+x_2+x_3) & \cos(x_1+x_2+x_3) \\ \sin(x_1+x_2+x_3) & \sqrt{2}+0.1 & \cos(x_1+x_2+x_3) \\ \cos(x_1+x_2+x_3) & \cos(x_1+x_2+x_3) & 2.1 \end{bmatrix}$$

(6 points)

Exercise 2. (weak formulation I)

Let $\Omega \subset \mathbb{R}^d$ be open and bounded, with smooth boundary $\partial\Omega$. Let $n_{\partial\Omega(x)}$ be the outer normal vector. For functions $u \in \mathcal{C}^4(\Omega)$, we consider the PDE

$$\begin{aligned} \Delta[\Delta u](x) &= f(x) \text{ in } \Omega \\ u(x) &= 0 \text{ on } \partial\Omega \\ \nabla u(x) \cdot n_{\partial\Omega}(x) &= 0 \text{ on } \partial\Omega\,. \end{aligned}$$

Derive the corresponding weak formulation in the space

$$H_0^2(\Omega) = \{u \in H^2(\Omega) \mid u(x) = 0 \text{ on } \partial\Omega,\ \nabla u(x)\cdot n_{\partial\Omega}(x) = 0 \text{ on } \partial\Omega\}\,.$$

(6 points)

Exercise 3. (higher regularity in 1D)

Let $I = [a,b] \subset \mathbb{R}$ and $f \in L^2(I)$. Let $u \in H_0^1(I)$ be the weak solution to the Poisson equation

$$-u'' = f$$

with Dirichlet boundary conditions. Show that u belongs to $H^2(I)$.

(4 points)